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Robustness and Predictivity of 4 TeV Unification

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Abstract

The stability of the predictions of two of the standard model parameters, $\alpha_3(M_Z)$ and $\sin^2\theta(M_Z)$, in a $M_U \sim 4$ TeV unification model is examined. It is concluded that varying the unification scale between $M_U \simeq 2.5$ TeV and $M_U \simeq 5$ TeV leaves robust all predictions within reasonable bounds. Choosing $M_U = 3.8 \pm 0.4$ TeV gives, at lowest order, accurate predictions at M_Z.

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One of the principal motivations for extending the standard model is the GUT gauge hierarchy between the weak scale and the grand unification or GUT scale. A related concern, not addressed here, is the Planck hierarchy between the weak scale and the Planck scale; the model we consider has flat spacetime, vanishing Newton's constant and infinite Planck scale.

The most popular solution of the GUT hierarchy is low-energy supersymmetry [1–4] where the three gauge couplings $\alpha_i(\mu)$ (i = 1,2,3) run logarithmically from $\mu = M_Z \sim$ 91 GeV, where they are known, up to $M_{GUT} \sim 2 \times 10^{16}$ GeV, where they coincide with impressive accuracy.

In a recently-proposed model [5], grand unification occurs differently. The three couplings run from $\mu = M_Z$ up to a lower unification scale $M_U \sim 4$ TeV, at which scale the theory is embedded in a larger gauge group $G \equiv SU(3)^{12}$. The $SU(3)$ gauge couplings $\alpha_j(\mu)$ (j=1-12) are all equal at $\mu = M_U$. The embedding of the standard model gauge group in the larger gauge group G provides a group-theoretical explanation for the different values of $\alpha_i(M_U)$.

This low-scale unification model also has a top-down inspiration from string theory through the AdS/CFT correspondence [6–8] arising from consideration of a Type IIB superstring in d = 10 dimensional spacetime compactified on $AdS_5 \times S^5$. Using a finite group $\Gamma = Z_{12}$ in an abelian orbifold $AdS_5 \times S^5/\Gamma$ gives a quiver gauge theory [9] with gauge group $SU(N)^{12}$ either with no supersymmetry $\mathcal{N} = 0$ [5] or with $\mathcal{N} = 1$ supersymmetry [10]

Several issues were left open in [5]: robustness of the predictions under variations of the scale M_U (conversely, the accuracy of the predictions at $\mu = M_Z$); the size of flavor-changing effects, and the consistency of the additional states around $M \sim M_U$ with constraints imposed by precision low-energy data. In this article we shall address all of these issues.

Robustness of Predictions to Variation in M_U

The calculations of [5] were done in the one-loop approximation to the renormalization group equations without threshold effects. Because the couplings remain weak this can be self-consistent provided the masses of the new states in the model are sufficiently close to M_U. Other corrections due to non-perturbative effects, and the effects of large extra dimensions, are outside of the scope of this paper. In one sense the robustness of this TeV-scale unification is almost self-evident, in that it follows from the weakness of the coupling constants in the evolution from M_Z to M_U. That is, in order to define the theory at M_U, one must combine the effects of threshold corrections (due to O($\alpha(M_U)$) mass splittings) and potential corrections from redefinitions of the coupling constants and the unification scale. We can then *impose* the coupling constant relations at M_U as renormalization conditions and this is valid to the extent that higher order corrections do not destabilize the vacuum state.

We shall approach the comparison with data in two different but almost equivalent ways. The first is "bottom-up", where we use as input the requirement that the values of $\alpha_3(\mu)/\alpha_2(\mu)$ and $\sin^2\theta(\mu)$ are expected to be 5/2 and 1/4, respectively, at $\mu = M_U$. Using the experimental ranges allowed for $\sin^2\theta(M_Z) = 0.23113 \pm 0.00015$, $\alpha_3(M_Z) = 0.1172 \pm 0.0020$ and $\alpha_{em}^{-1}(M_Z) = 127.934 \pm 0.027$ from [11] we have plotted in Figure 1 the values of $\sin^2\theta(M_U)$ (vertical axis) and $\alpha_3(M_U)/\alpha_2(M_U)$ (horizontal axis) for a range of M_U between 1.5 TeV and 8 TeV. Allowing a maximum discrepancy of $\pm 1\%$ in $\sin^2\theta(M_U)$ and $\pm 4\%$ in $\alpha_3(M_U)/\alpha_2(M_U)$ as reasonable estimates of corrections, we deduce that the unification scale M_U may vary between 2.5 TeV and 5 TeV. Thus the theory is robust in the sense that uncertainty in the renormalization group equations does not effect the existence of unification.

Accuracy of Predictions at $\mu = M_Z$

Alternatively, to test of predictivity we fix the unification values at M_U of $\sin^2\theta(M_U) = 1/4$ and $\alpha_3(M_U)/\alpha_2(M_U) = 5/2$ and compute the resultant predictions at the scale $\mu = M_Z$. The results are shown for $\sin^2\theta(M_Z)$ in Fig. 2 with the allowed range [11] $\alpha_3(M_Z) = 0.1172 \pm 0.0020$. The precise data on $\sin^2(M_Z)$ are indicated in Fig. 2 demonstrating that the model makes correct predictions for $\sin^2\theta(M_Z)$. Similarly, in Fig 3, there is a plot of the prediction for $\alpha_3(M_Z)$ versus M_U with $\sin^2\theta(M_Z)$ held within the allowed empirical range. The two quantities plotted in Figs 2 and 3 are consistent for similar ranges of M_U: both $\sin^2\theta(M_Z)$ and $\alpha_3(M_Z)$ are within the empirical limits if $M_U = 3.8 \pm 0.4$ TeV.

Discussion

The model has many additional gauge bosons at the unification scale, including neutral Z''s and charged $W's$, which could mediate flavor-changing processes on which there are strong empirical upper limits. The lower bound on a Z' coupling like the standard Z is $M(Z') < 1.5$ TeV [11] which is below the M_U values considered here; however, the couplings of the other $SU(3)$ gauge groups associated with $SU(3)_W$ have a coupling generically stronger by a factor 4 requiring that $M(Z'') < 6TeV$ and hence a real danger of too-strong FCNC. This is, in our view, the tightest constraint on the viability of such conformality models. Full analysis requires commitment to a specific identification of quark flavors in the quiver diagram .

Since there are many new states predicted at the unification scale ~ 4 TeV, there is, in addition, a potential of being ruled out by other precision low energy data, as conveniently studied in terms of the parameters S and T introduced in [12], designed to measure departure from the predictions of the standard model. Concerning T, if the new $SU(2)$ doublets are mass-degenerate and hence do not violate a custodial $SU(2)$ symmetry, they do not contribute T. This provides a constraint on the spectrum of new states. According to [12], a multiplet of degenerate heavy chiral fermions gives a contribution to S:

$$S = C \sum_i \left(t_{3L}(i) - t_{3R}(i)\right)^2 /3\pi \tag{1}$$

where $t_{3L,R}$ is the third component of weak isopspin of the left- and right- handed component of fermion i and C is the number of colors. In the present model, the additional fermions are non-chiral and fall into vector-like multiplets and so do not contribute to S. Provided that the extra isospin multiplets at the unification scale M_U are sufficiently mass-degenerate, therefore, there is no conflict of chiral fermions with precision data at low energy.

For contribution of new gauge bosons, we refer to the analysis in [13]. In the limit where the bilepton gauge bosons are degenerate $M_{++} = M_+$ the contribution to S vanishes except for the subtlety of the pinch contribution. From the formula presented in [13] we find ($S|_P$ is the pinch contribution):

$$S = S_0 + S|_P \tag{2}$$

The first term in Eq.(2) is explicitly:

$$\begin{aligned}
S_0 &= -16\pi \mathrm{Re} \frac{\Pi^{3Y}(m_Z^2) - \Pi^{3Y}(0)}{m_Z^2} \\
&= \frac{9}{4\pi}\Bigg[\ln\frac{M_{++}^2}{M_+^2} + \frac{2}{m_Z^2}\left(M_{++}^2 \bar{F}_0(m_Z^2, M_{++}, M_{++}) - M_+^2 \bar{F}_0(m_Z^2, M_+, M_+)\right) \\
&\quad + \frac{4}{3}\left(\bar{F}_0(m_Z^2, M_{++}, M_{++}) - \bar{F}_0(m_Z^2, M_+, M_+)\right) \\
&\quad - 2\left(\bar{F}_3(m_Z^2, M_{++}, M_{++}) - \bar{F}_3(m_Z^2, M_+, M_+)\right)\Bigg],
\end{aligned} \tag{3}$$

in which $\bar{F}_{0,3}$ are given by:

$$\begin{aligned}
\bar{F}_0(s, M, m) &= \int_0^1 dx\, \ln\Big((1-x)M^2 + xm^2 - x(1-x)s\Big) - \ln Mm \\
&= \tfrac{2}{s}\sqrt{(M+m)^2 - s}\sqrt{s - (M-m)^2}\tan\sqrt{\tfrac{s-(M-m)^2}{(M+m)^2-s}} + \tfrac{M^2-m^2}{s}\ln\tfrac{M}{m} - 2,
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\bar{F}_3(s, M, m) &= \int_0^1 dx\, x(1-x)\ln\Big((1-x)M^2 + xm^2 - x(1-x)s\Big) - \frac{1}{6}\ln Mm \\
&= \frac{1}{6}\left[1 + \frac{M^2+m^2}{s} - \frac{2(M^2-m^2)^2}{s^2}\right]\bar{F}_0(s, M, m) \\
&\quad - \frac{1}{6}\left(1 - \frac{2(M^2+m^2)}{s}\right)\frac{M^2-m^2}{s}\ln\frac{M}{m} + \frac{1}{18} - \frac{(M^2-m^2)^2}{3s^2}.
\end{aligned} \tag{5}$$

The second term in Eq.(2) is:

$$\begin{aligned}
S|_P &= \frac{1}{\pi}\Bigg[3\ln\frac{M_{++}^2}{M_+^2} + 2(1 + 2\sin^2\theta_W)\bar{F}_0(m_Z^2, M_{++}, M_{++}) \\
&\quad - (1 - 4\sin^2\theta_W)\bar{F}_0(m_Z^2, M_+, M_+)\Bigg].
\end{aligned} \tag{6}$$

From these equations, we find that the contributions of gauge bosons to S are suppressed by $(M_Z/M_U)^2 \sim 10^{-4}$ and so even for many such new gauge bosons the contribution to S is acceptably small provided the $SU(2)$ doublets are adequately degenerate.

The plots we have presented clarify the accuracy of the predictions of this TeV unification scheme for the precision values accurately measured at the Z-pole. The predictivity is as accurate for $\sin^2\theta$ as it is for supersymmetric GUT models [1–4]. There is, in addition, an accurate prediction for α_3 which is used merely as input in SusyGUT models.

At the same time, the accuracy of the predictions remains robust if we allow the unification scale to vary from about 2.5 TeV to 5 TeV.

In conclusion, since this model ameliorates the GUT hierarchy problem and naturally accommodates three families, it provides a viable alternative to the widely-studied GUT models which unify by logarithmic evolution of couplings up to much higher scales.

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Figure Captions

Fig. 1.

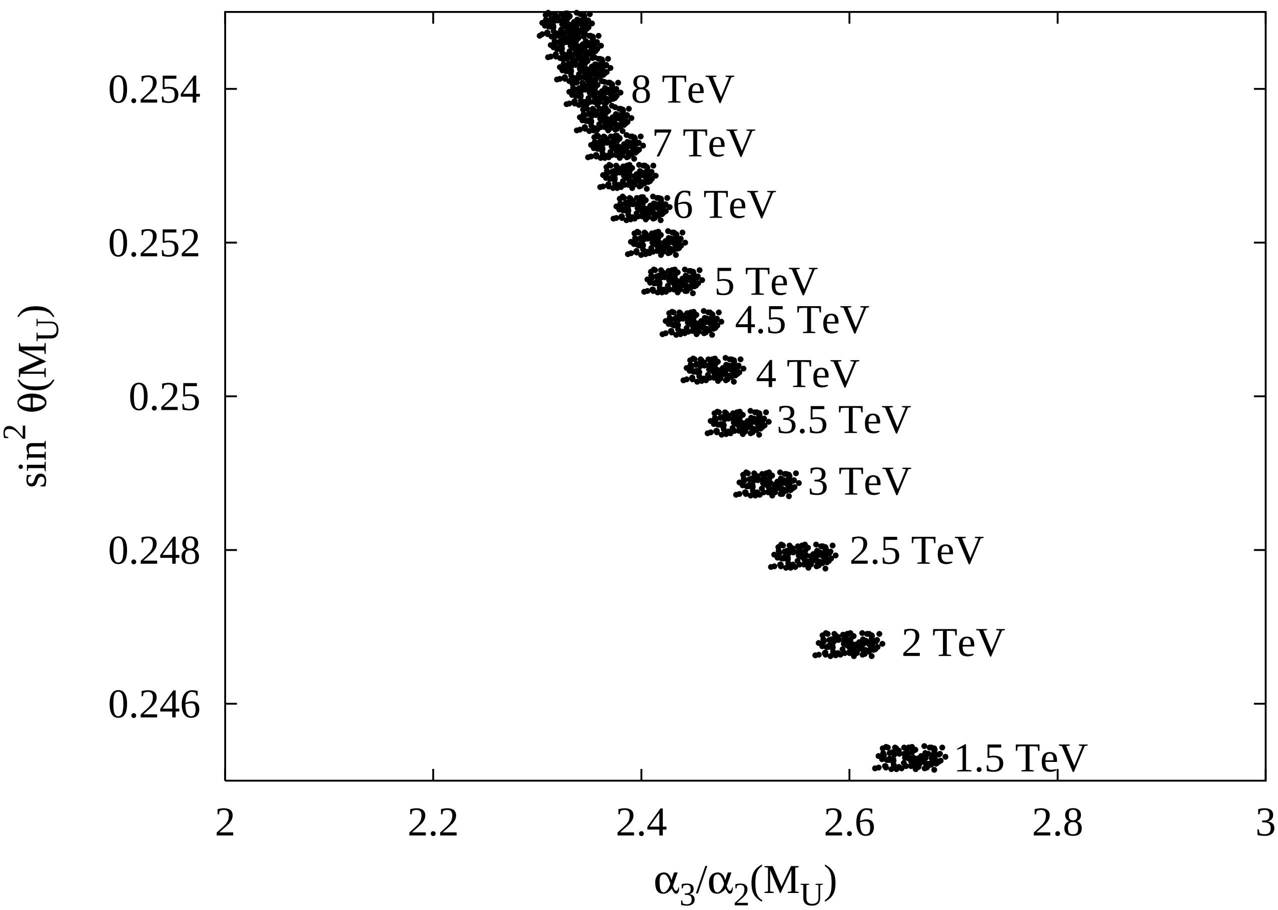
Plot of $\sin^2\theta(M_U)$ versus $\alpha_3(M_U)/\alpha_2(M_U)$ for various choices of M_U.

Fig.2.

Plot of $\sin^2\theta(M_Z)$ versus M_U in TeV, assuming $\sin^2\theta(M_U) = 1/4$ and $\alpha_3/\alpha_2(M_U) = 5/2$.

Fig.3.

Plot of $\alpha_3(M_Z)$ versus M_U in TeV, assuming $\sin^2\theta(M_U) = 1/4$ and $\alpha_3/\alpha_2(M_U) = 5/2$.



8 TeV
7 TeV
6 TeV
5 TeV
4.5 TeV
4 TeV
3.5 TeV
3 TeV
2.5 TeV
2 TeV
1.5 TeV
0.254
0.252
0.25
0.248
0.246
$\sin^2\theta(M_U)$
2
2.2
2.4
2.6
2.8
3
$\alpha_3/\alpha_2(M_U)$

FIG. 1.

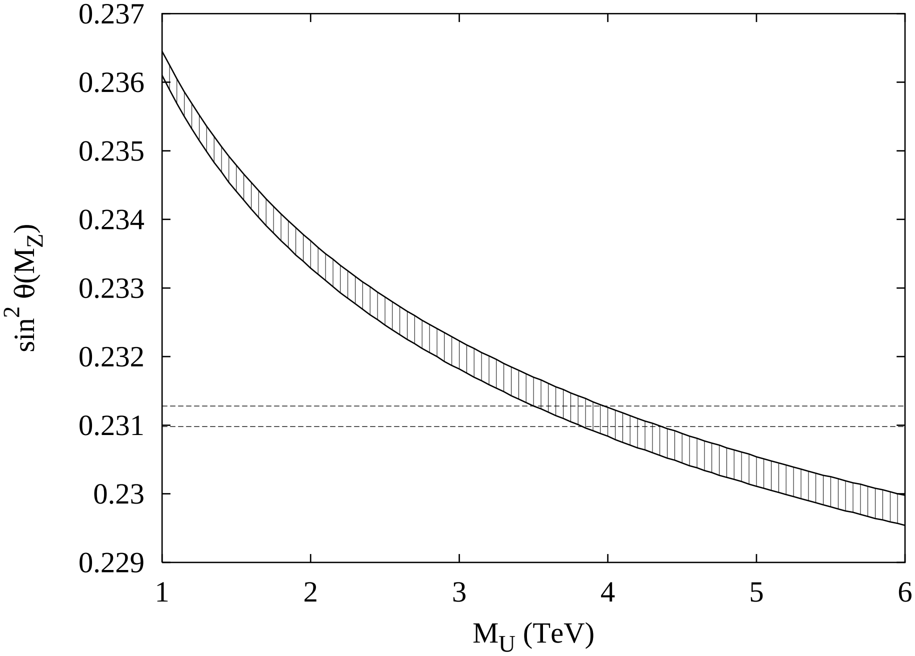

0.237
0.236
0.235
0.234
0.233
0.232
0.231
0.23
0.229
$\sin^2\theta(M_Z)$
1
2
3
4
5
6
M_U (TeV)

FIG. 2.

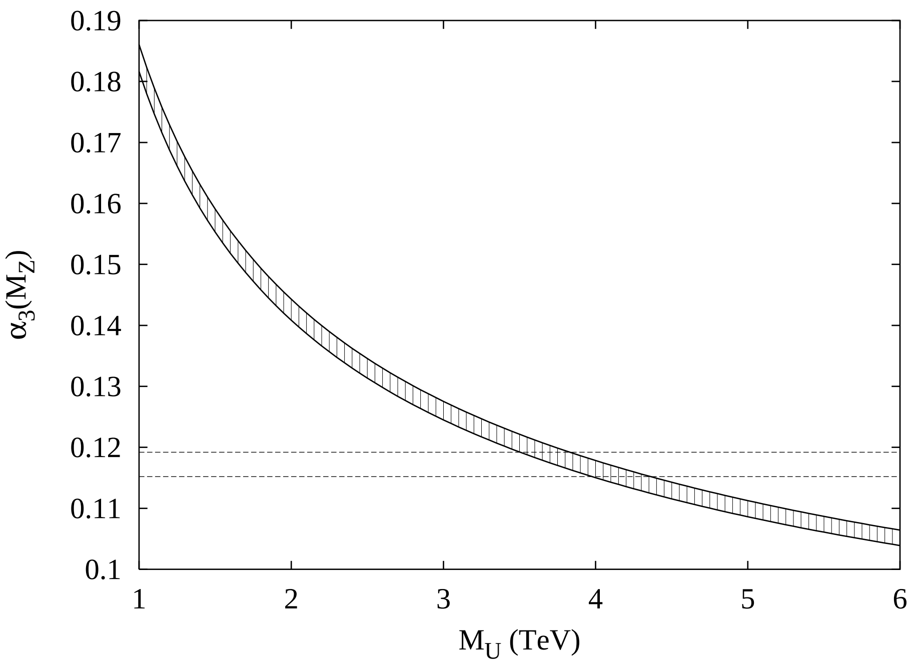

0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
$\alpha_3(M_Z)$
1
2
3
4
5
6
M_U (TeV)

FIG. 3.